

# Ideal MOS Curves for Silicon

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(Manuscript received April 22, 1966)

*Ideal curves of MOS capacity and surface potential are computed for silicon with oxide thickness and doping as a parameter. High-frequency and low-frequency capacity curves are presented for the doping ranges between  $1 \times 10^{14}$  and  $1 \times 10^{17} \text{ cm}^{-3}$  and  $\text{SiO}_2$  thickness between 100 and 12000 Å. Additional curves give flatband capacitance, minimum capacitance and voltage of the minimum capacitance in the same ranges.\**

Measurement of MOS (Metal-Oxide-Semiconductor) capacity vs voltage is a convenient and widely used technique for evaluation of the properties of semiconductor-insulator interfaces.<sup>1,2,3</sup> Most MOS-type measurements require comparison of the experimentally measured curve with the ideal curve. This is usually time consuming because the ideal curve is represented by rather complicated functions and has to be calculated for each value of silicon doping and oxide thickness separately.

The purpose of this work is to facilitate such evaluations by providing a collection of curves embracing all the practically important ranges.

Three sets of curves are furnished.

- (i) Plots of differential capacity vs voltage with thickness as a parameter and P-doping density  $N_A$  varying from plot to plot. The capacity is normalized with respect to the oxide capacity  $C_{ox}$ . The dashed lines are the high-frequency branch of the curves.
- (ii) A plot of surface potential  $\psi_s$  corresponding to each of the preceding curves vs applied voltage  $V$ . For definition of the surface potential see below.
- (iii) A set of curves giving a survey of pertinent results of this calculation.
  - (a) Flatband capacity  $C_{FB}$  (defined as capacity for  $\psi_s = 0$ ) vs oxide thickness with doping density as parameter.
  - (b) The minimum capacitance  $C_{min}/C_{ox}$  plotted in the same manner.

\* Additional curves covering intermediate doping ranges of  $2 \times 10^{14} \text{ cm}^{-3}$ ,  $5 \times 10^{14}$ , etc., are available on request.

- (c) Voltage at which the minimum capacitance occurs. This is the voltage difference between the point of flatband capacity and minimum capacity.

The entire calculation was done on the IBM 7094 computer. The low-frequency curves were computed for room temperature using the well-known<sup>1,4,5,6</sup> relations between capacitance, surface potential, and applied voltage. Pertinent constants used were: dielectric permittivity of silicon;  $\epsilon_{\text{Si}} = 1.06 \times 10^{-12}$  F/cm, dielectric permittivity of  $\text{SiO}_2$ ;  $\epsilon_{\text{ox}} = 3.4 \times 10^{-13}$  F/cm. The calculation was carried out for *p*-type silicon and  $\text{SiO}_2$  insulator, but the results can be adapted to *n*-type silicon and insulators other than  $\text{SiO}_2$ . The conversion to *n*-type is achieved simply by changing the sign of the voltage axis, the conversion to other insulators requires scaling of the oxide thickness with the ratio of the dielectric constants of  $\text{SiO}_2$  and other insulator

$$d_c = d_a \frac{\epsilon_{\text{ox}}}{\epsilon_D}$$

where  $d_c$  = thickness to be used in these curves,  $d_a$  = actual thickness,  $\epsilon_D$  = dielectric permittivity of the new dielectric,  $\epsilon_{\text{ox}} = 3.4 \times 10^{-13}$  F/cm. Surface potential  $\psi_s$  as used here is defined as the amount of band bending from the bulk to the interface in volts.  $\psi_s$  is positive, when the bands are bent downwards from the flatband position. It should be noted that the origin of  $\psi_s$  is the flatband potential and that its distance from the band edges varies with doping.

The low-frequency curves were calculated under the assumption that minority carriers contribute fully to the capacity. For calculations of the high-frequency capacity, an approximation developed by Sah<sup>7</sup> was used. In this approximation, the space charge region is divided into three parts and their contribution to the capacity separately computed. The high-frequency inversion range was connected with the low-frequency curve by a smooth line.

The curves were computed for doping densities from  $1 \times 10^{14}$  to  $5 \times 10^{17}$   $\text{cm}^{-3}$  and oxide thicknesses from 100 to 12000 Å.

For the higher doping ranges, the computation was terminated at an oxide thickness giving a minimum capacitance of about 0.9 of the oxide capacitance. In some of the curves at lower doping overlapping ranges of oxide thickness were plotted in separate graphs to prevent the curves from coming too close to each other. For instance, one plot contains oxide thicknesses 600 Å, 1000 Å, etc., the other 800 Å, 1200 Å, etc.

## ACKNOWLEDGMENT

The help of R. S. Reichert with computer plotting routines is gratefully acknowledged.

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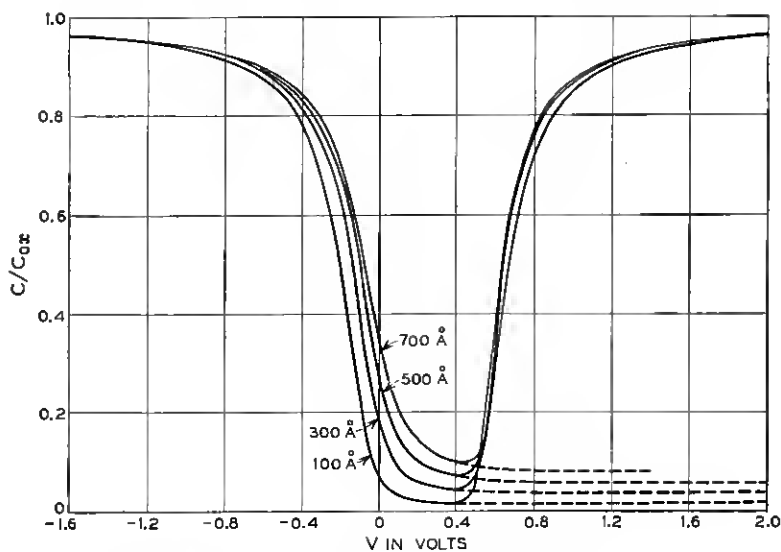


Fig. 1 — MOS capacity vs voltage. Oxide thickness 100-700 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

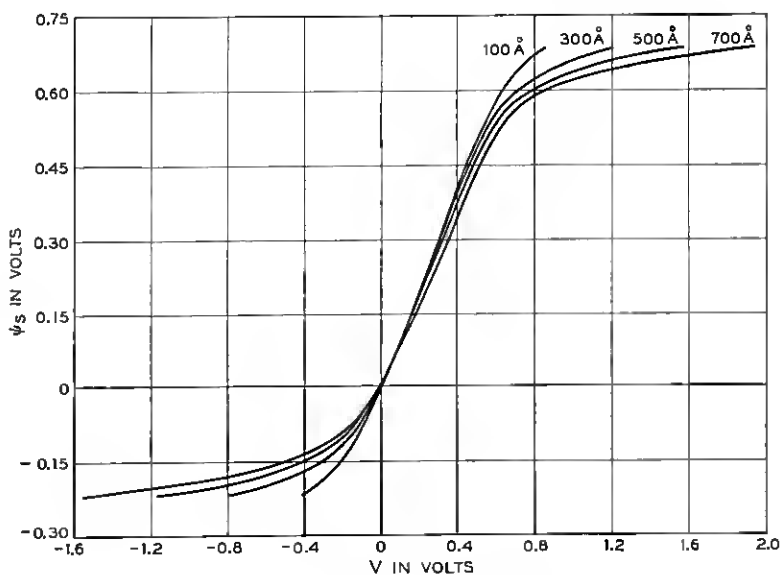


Fig. 2 — Surface potential vs voltage. Oxide thickness 100-700 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

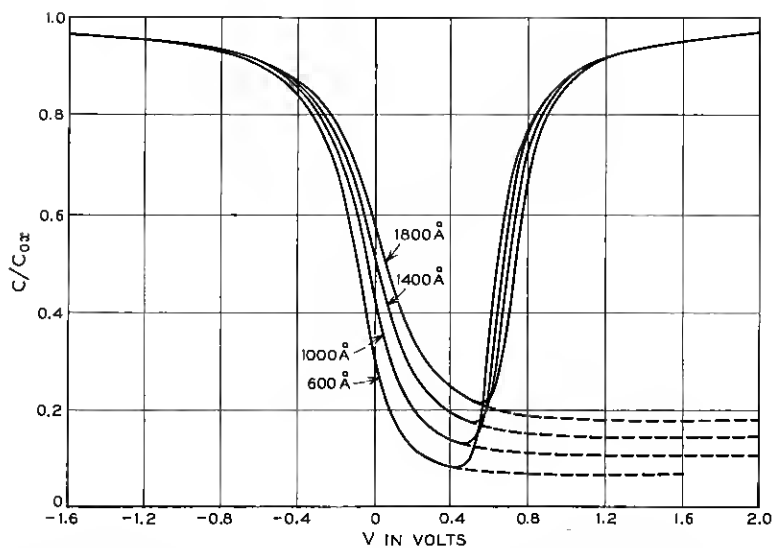


Fig. 3 — MOS capacity vs voltage. Oxide thickness 600–1800 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

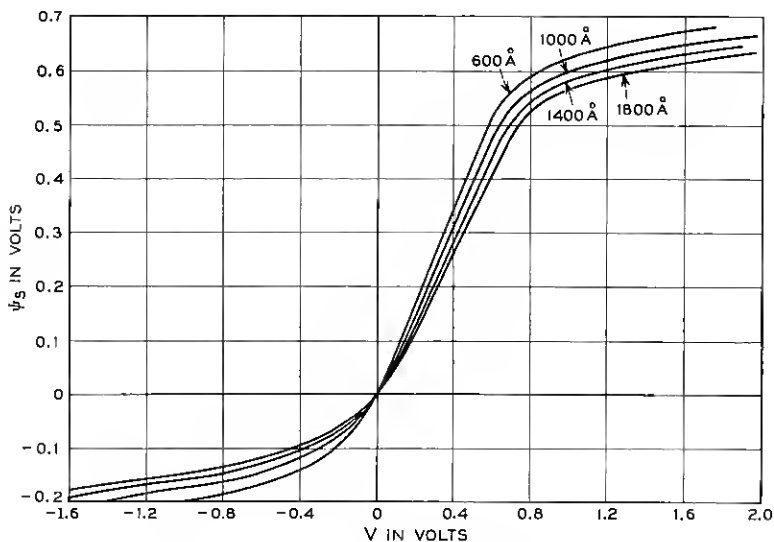


Fig. 4 — Surface potential vs voltage. Oxide thickness 600–1800 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

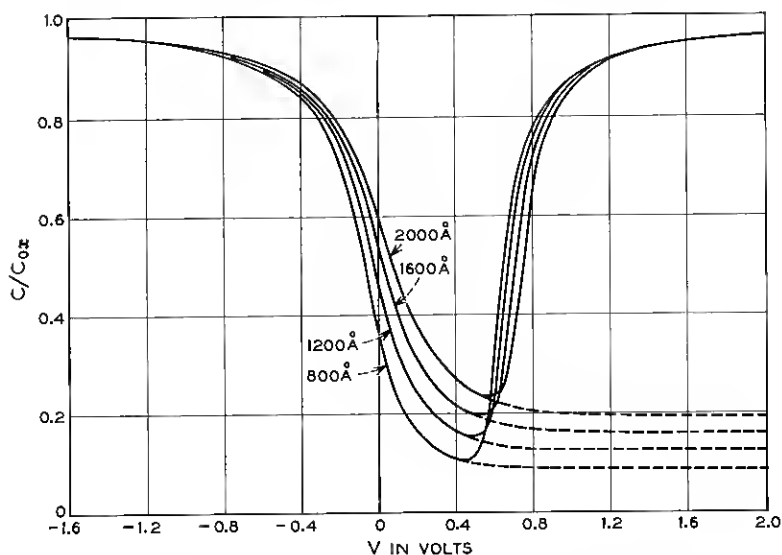


Fig. 5 — MOS capacity vs voltage. Oxide thickness 800-2000 Å ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

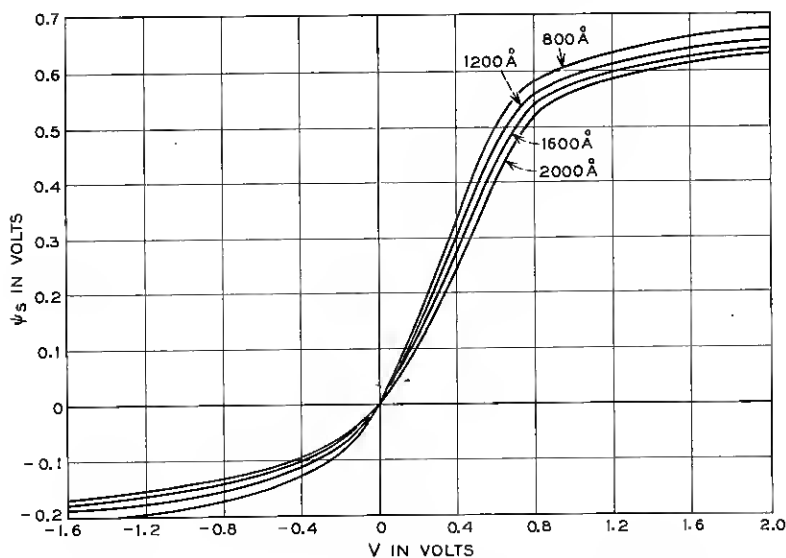


Fig. 6 — Surface potential vs voltage. Oxide thickness 800-2000 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

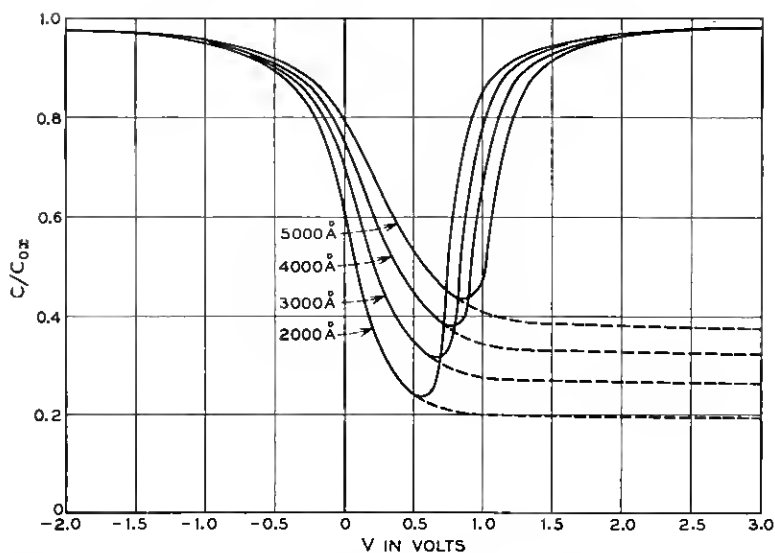


Fig. 7 — MOS capacity vs voltage. Oxide thickness 2000–5000 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

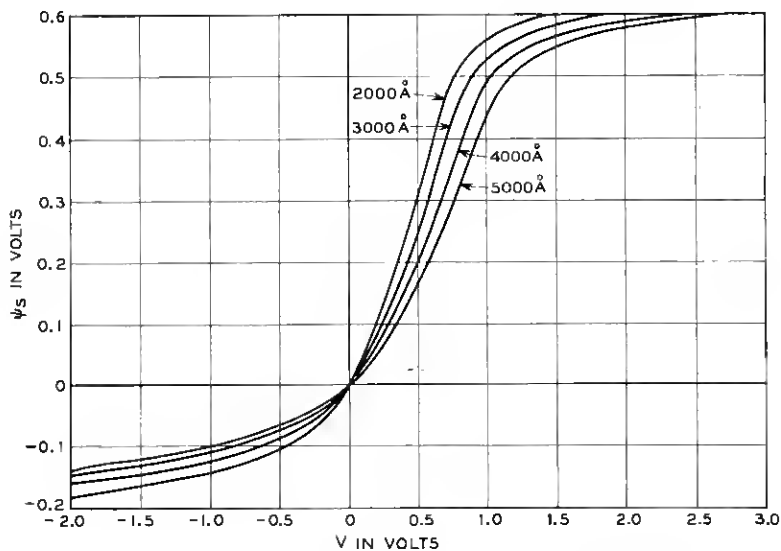


Fig. 8 — Surface potential vs voltage. Oxide thickness 2000–5000 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

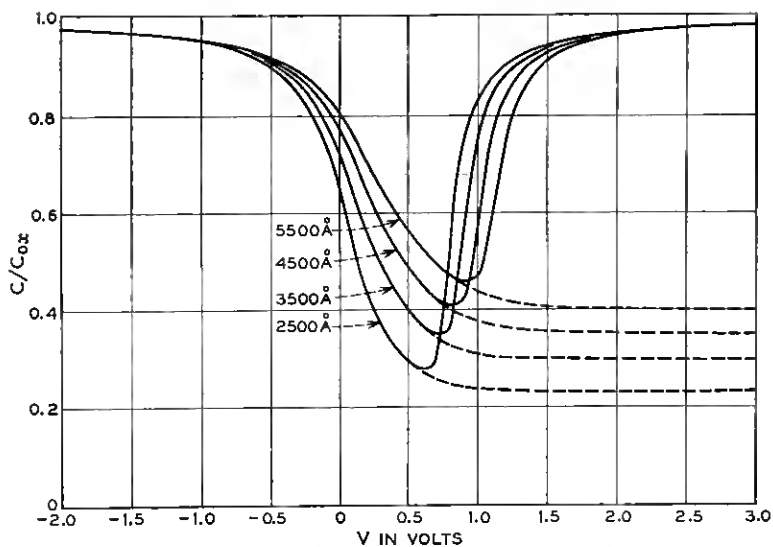


Fig. 9 — MOS capacity vs voltage. Oxide thickness 2500–5500 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

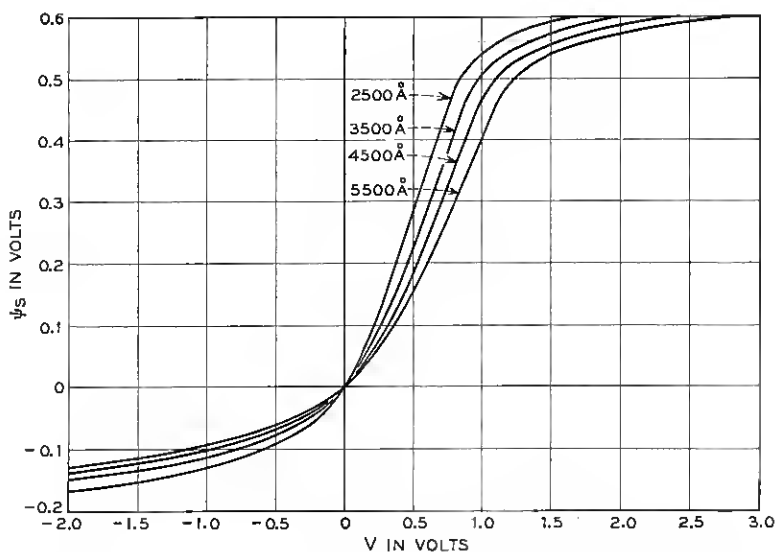


Fig. 10 — Surface potential vs voltage. Oxide thickness 2500–5500 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )



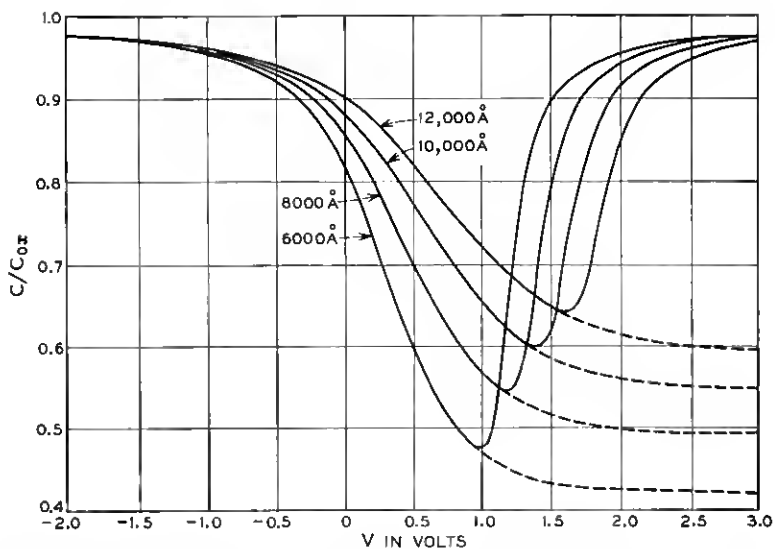


Fig. 11 — MOS capacity vs voltage. Oxide thickness 6000–12,000 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

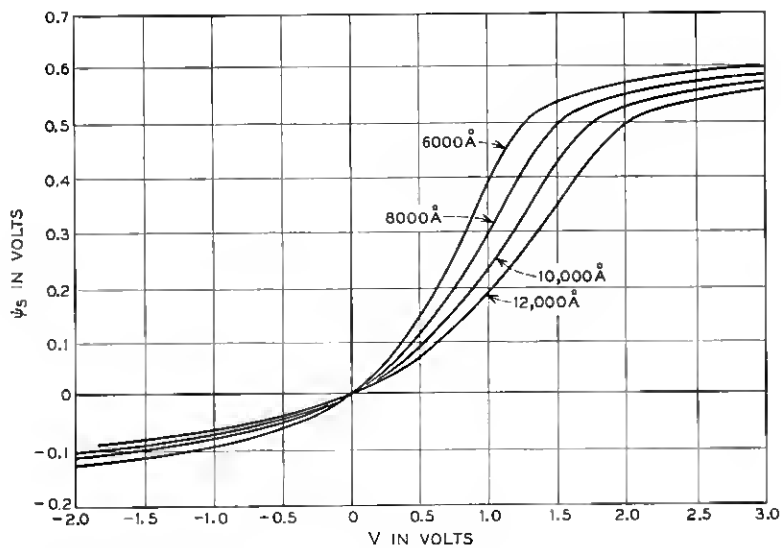


Fig. 12 — Surface potential vs voltage. Oxide thickness 6000–12,000 Å. ( $N_A = 1.0 \times 10^{14} \text{ cm}^{-3}$ )

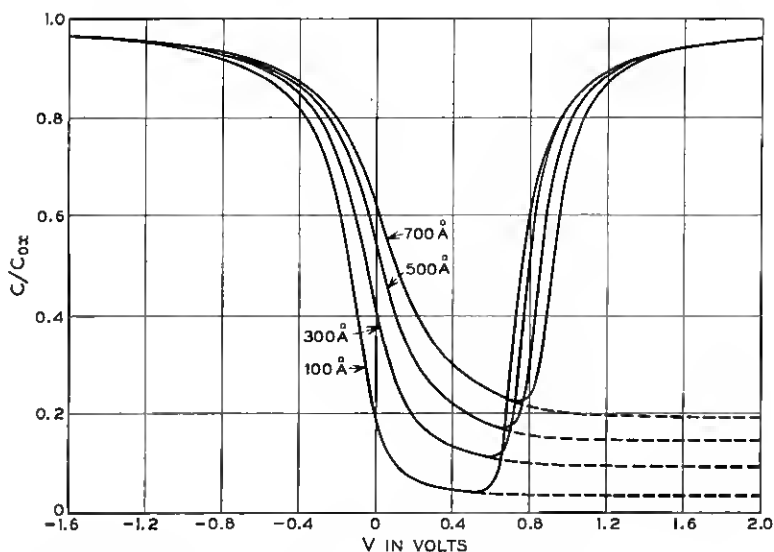


Fig. 13 — MOS capacity vs voltage. Oxide thickness 100–700 Å. ( $N_A = 1.0 \times 10^{18} \text{ cm}^{-3}$ )

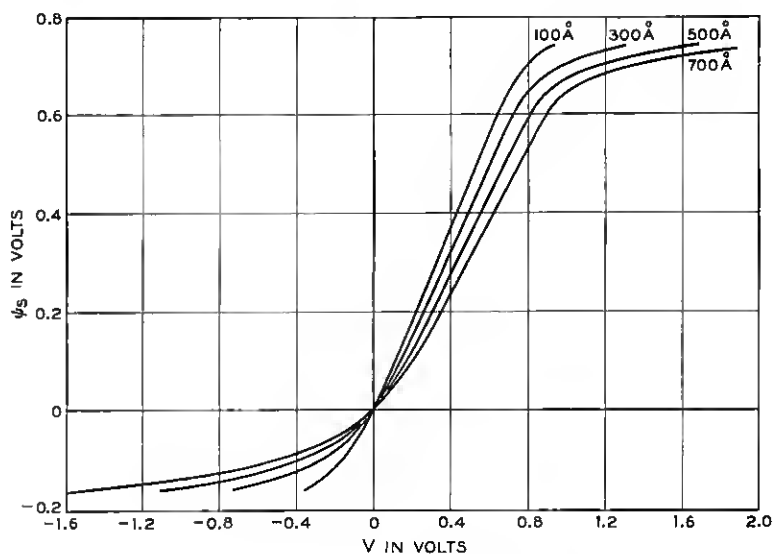


Fig. 14 — Surface potential vs voltage. Oxide thickness 100–700 Å. ( $N_A = 1.0 \times 10^{18} \text{ cm}^{-3}$ )

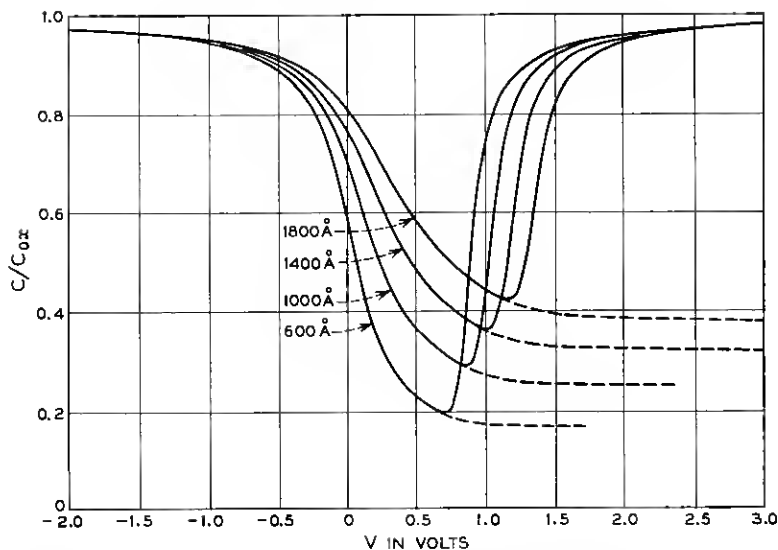


Fig. 15 — MOS capacity vs voltage. Oxide thickness 600–1800 Å. ( $N_A = 1.0 \times 10^{15} \text{ cm}^{-3}$ )

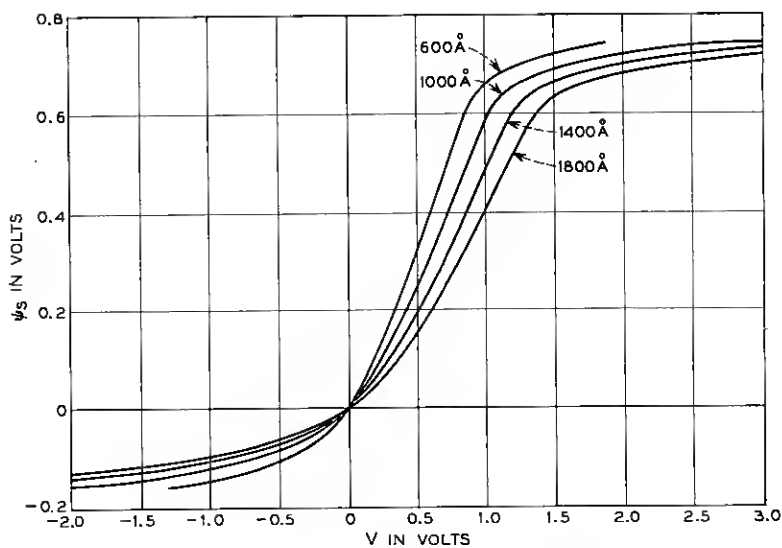


Fig. 16 — Surface potential vs voltage. Oxide thickness 600–1800 Å. ( $N_A = 1.0 \times 10^{15} \text{ cm}^{-3}$ )

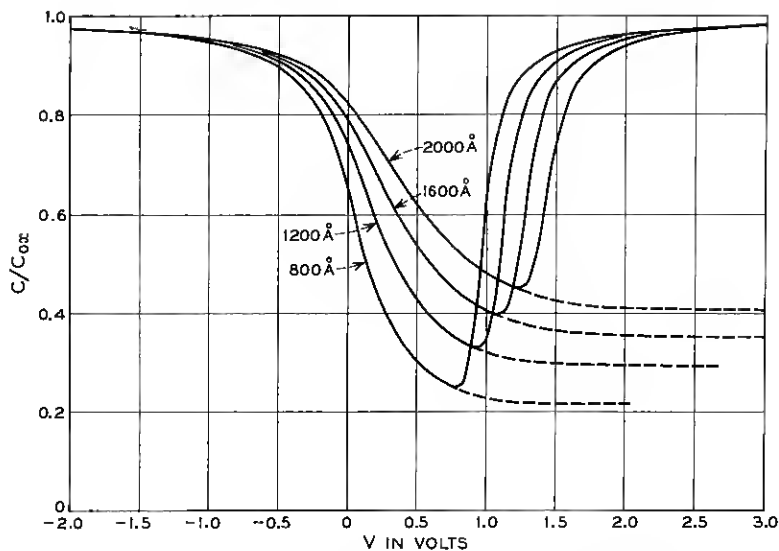


Fig. 17 — MOS capacity vs voltage. Oxide thickness 800–2000 Å. ( $N_A = 1.0 \times 10^{15} \text{ cm}^{-3}$ )

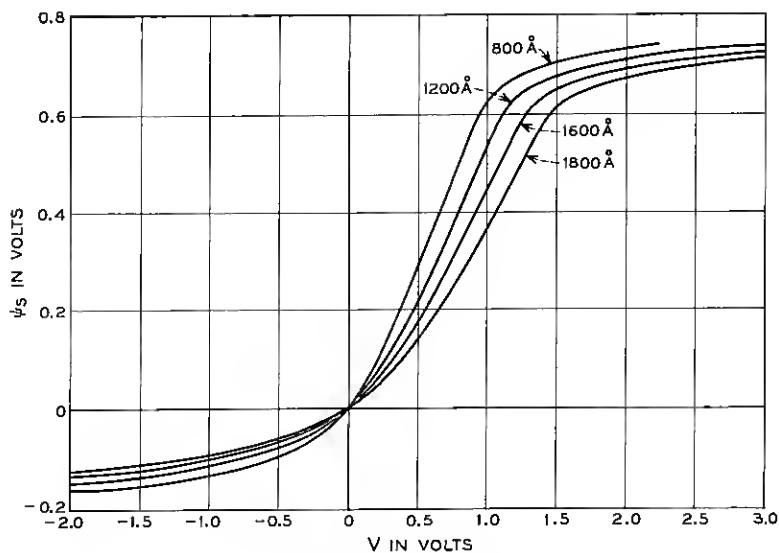


Fig. 18 — Surface potential vs voltage. Oxide thickness 800–2000 Å. ( $N_A = 1.0 \times 10^{15} \text{ cm}^{-3}$ )

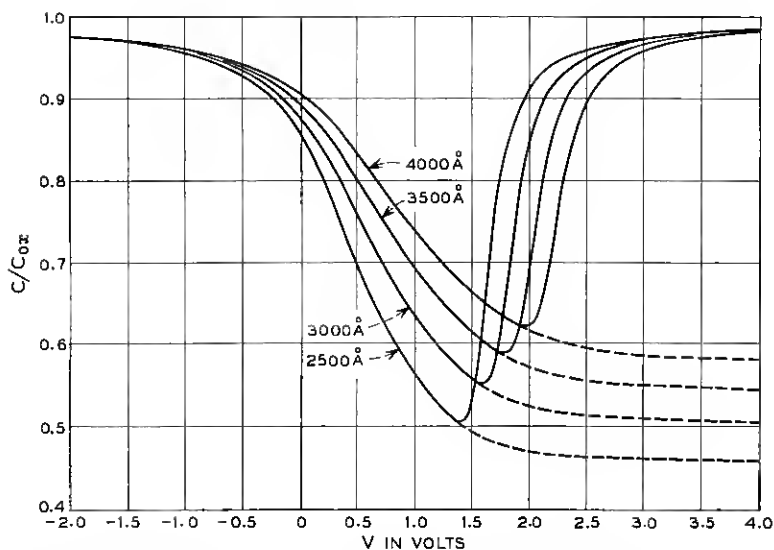


Fig. 19 — MOS capacity vs voltage. Oxide thickness 2500–4000 Å. ( $N_A = 1.0 \times 10^{15} \text{ cm}^{-3}$ )

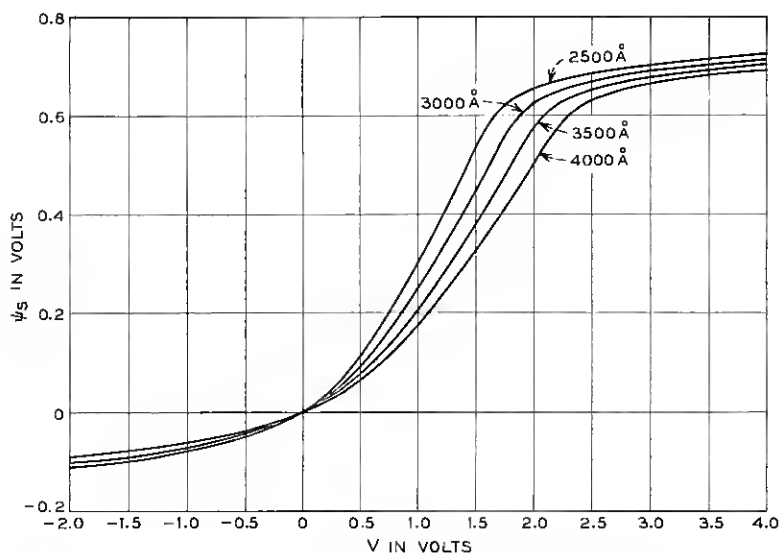


Fig. 20 — Surface potential vs voltage. Oxide thickness 2500–4000 Å. ( $N_A = 1.0 \times 10^{15} \text{ cm}^{-3}$ )

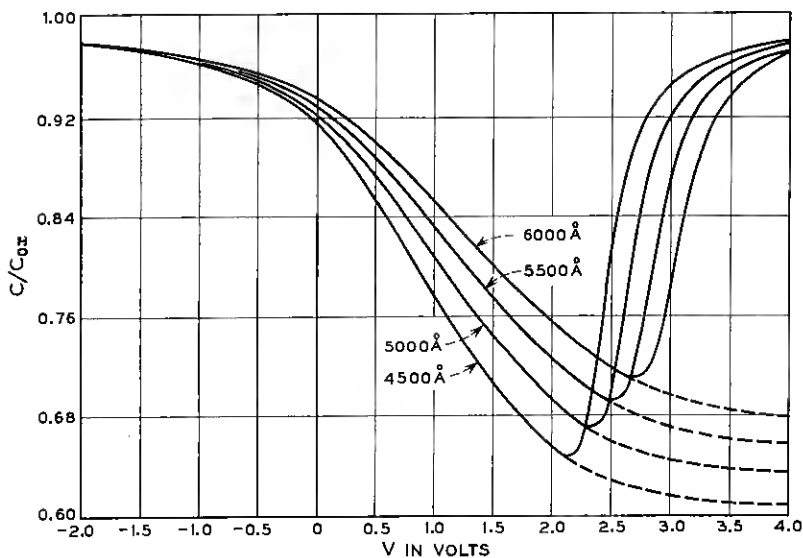


Fig. 21 — MOS capacity vs voltage. Oxide thickness 4500–6000 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

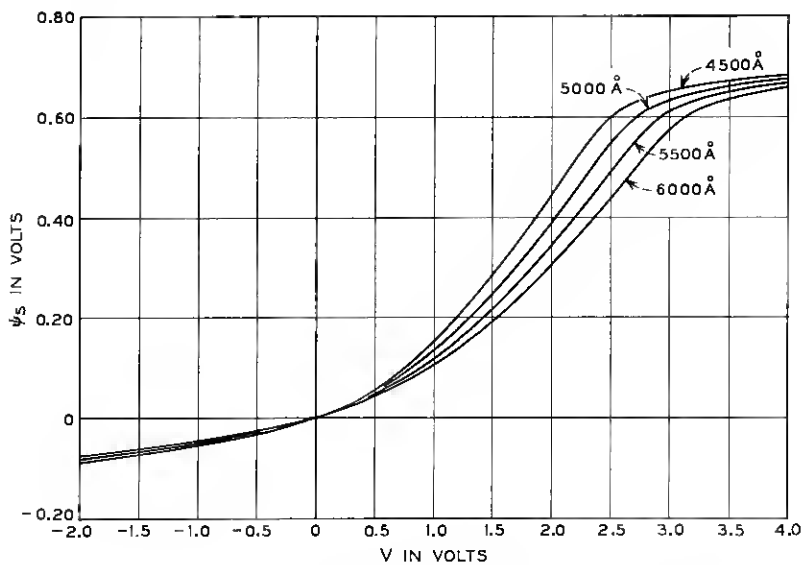


Fig. 22 — Surface potential vs voltage. Oxide thickness 4500–6000 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

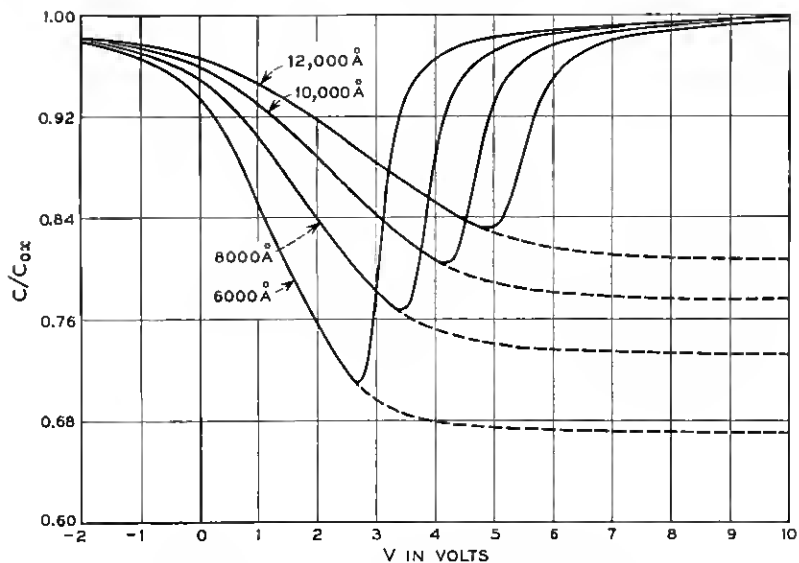


Fig. 23 — MOS capacity vs voltage. Oxide thickness 6000–12,000 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

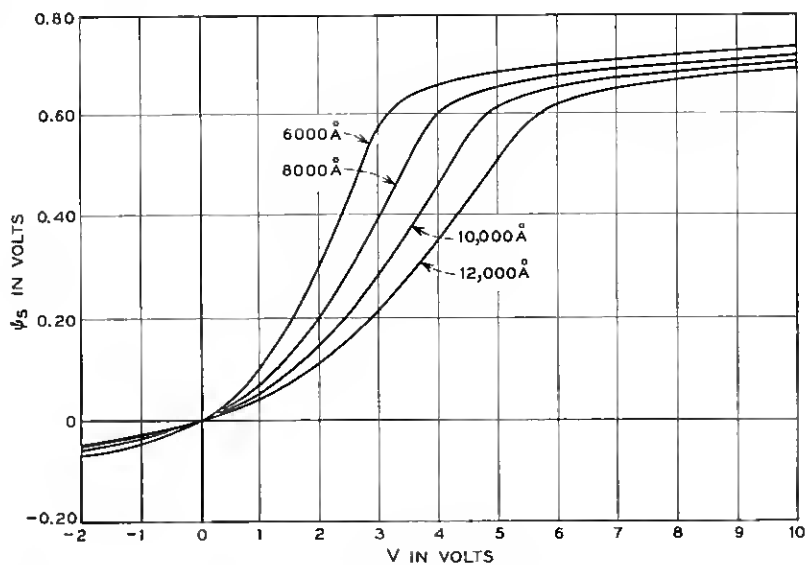


Fig. 24 — Surface potential vs voltage. Oxide thickness 6000–12,000 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

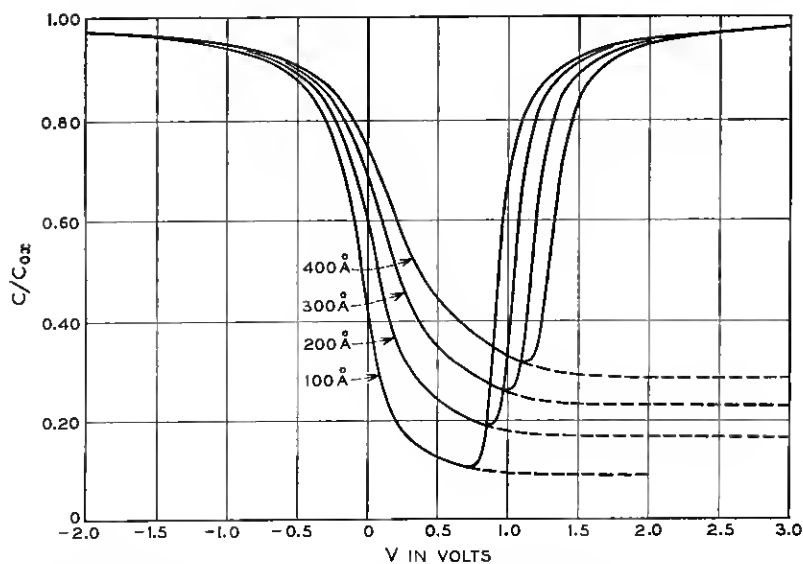


Fig. 25 — MOS capacity vs voltage. Oxide thickness 100–400 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

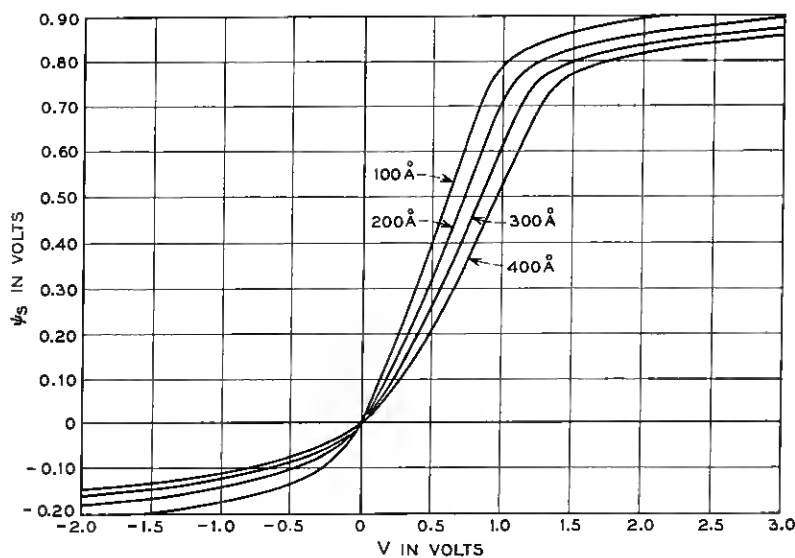


Fig. 26 — Surface potential vs voltage. Oxide thickness 100–400 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )



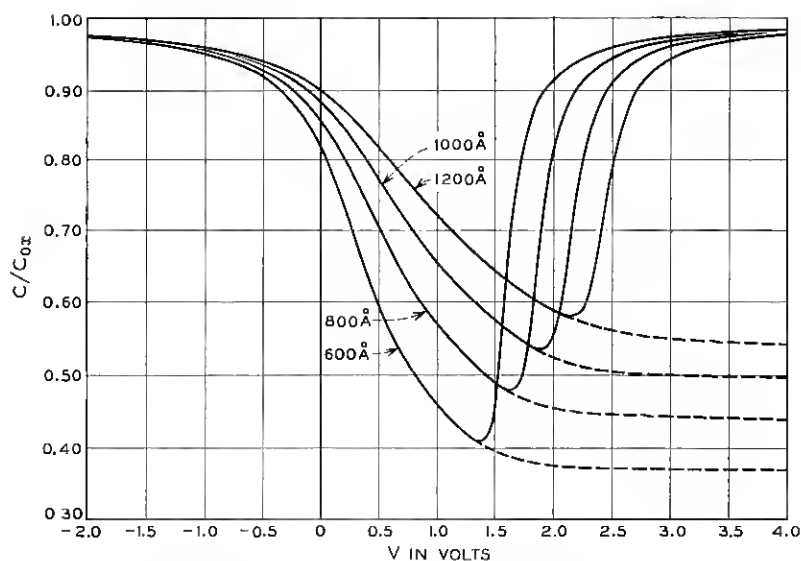


Fig. 27 — MOS capacity vs voltage. Oxide thickness 600–1200 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

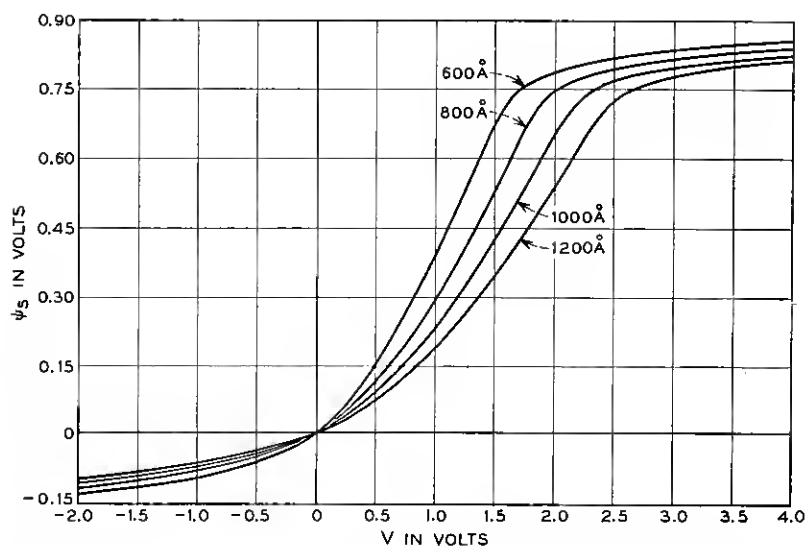


Fig. 28 — Surface potential vs voltage. Oxide thickness 600–1200 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

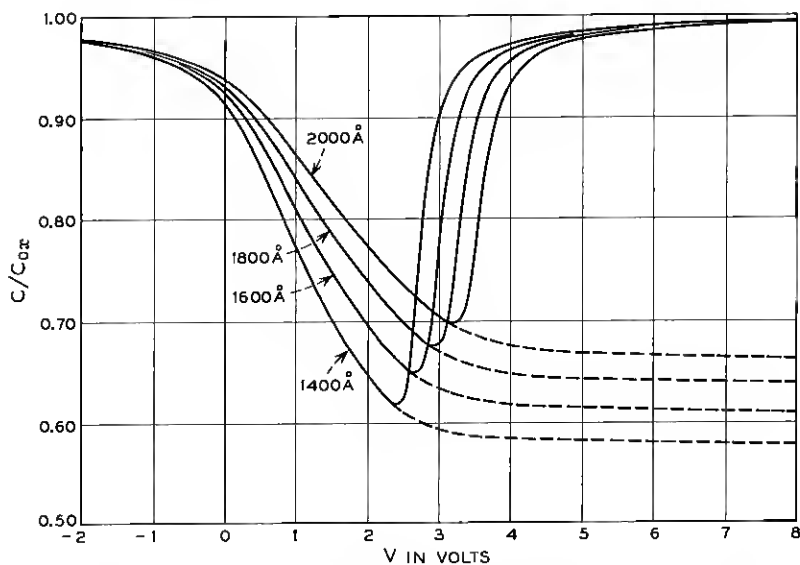


Fig. 29 — MOS capacity vs voltage. Oxide thickness 1400–2000 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

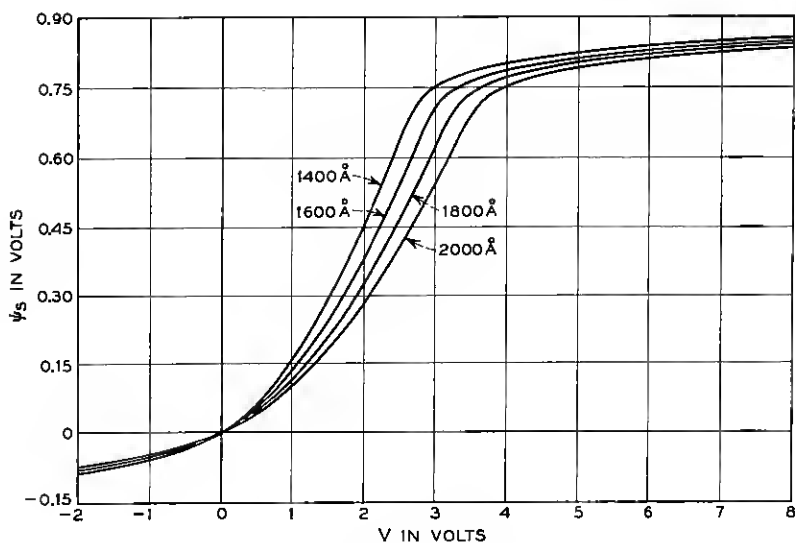


Fig. 30 — Surface potential vs voltage. Oxide thickness 1400–2000 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

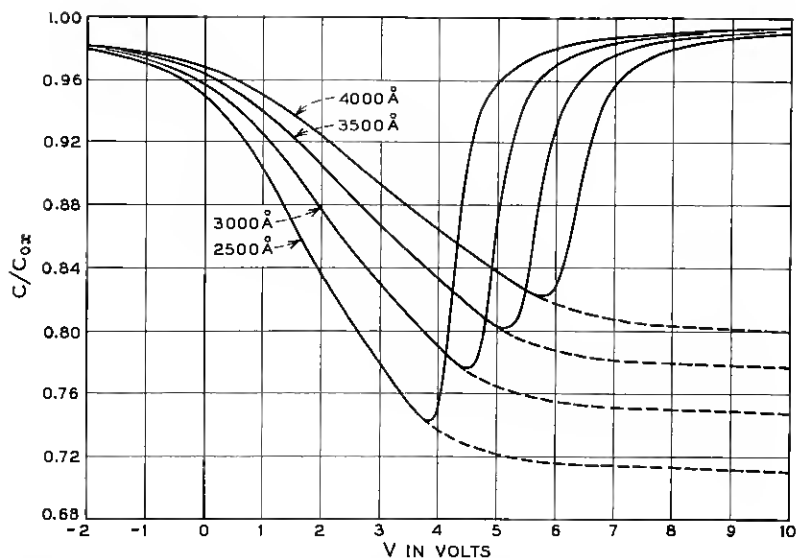


Fig. 31 — MOS capacity vs voltage. Oxide thickness 2500–4000 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

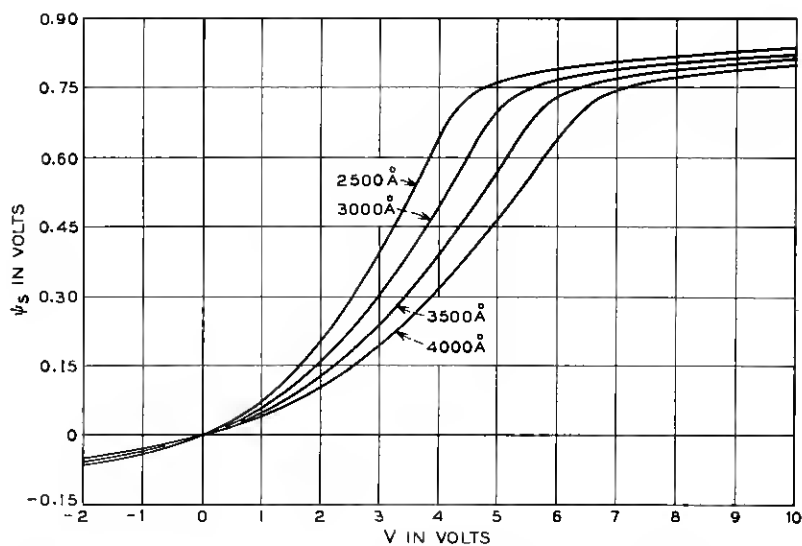


Fig. 32 — Surface potential vs voltage. Oxide thickness 2500–4000 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

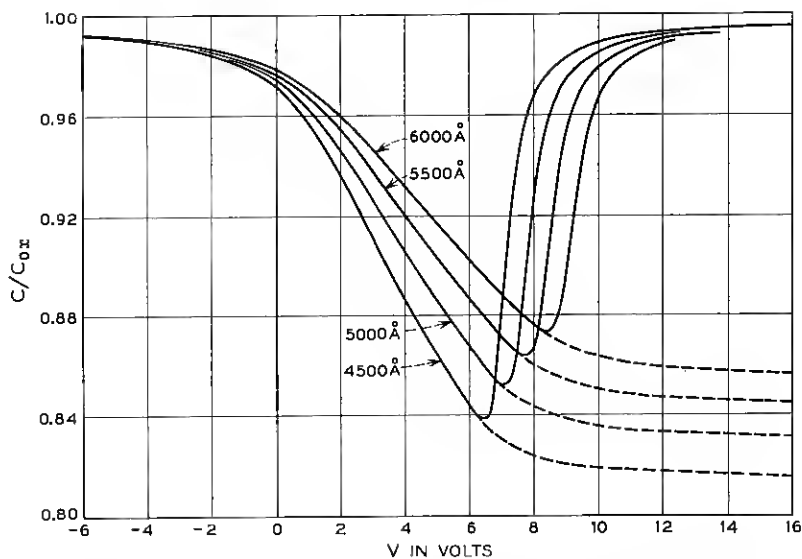


Fig. 33 — MOS capacity vs voltage. Oxide thickness 4500–6000 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

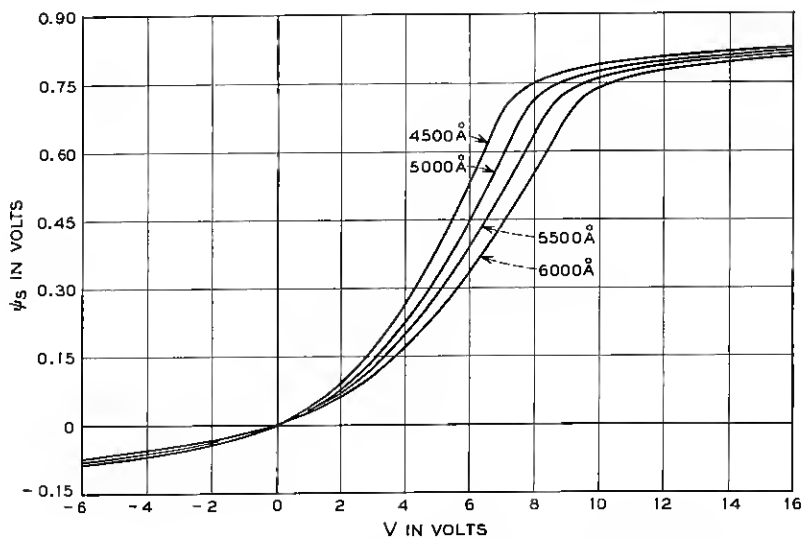


Fig. 34 — Surface potential vs voltage. Oxide thickness 4500–6000 Å. ( $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ )

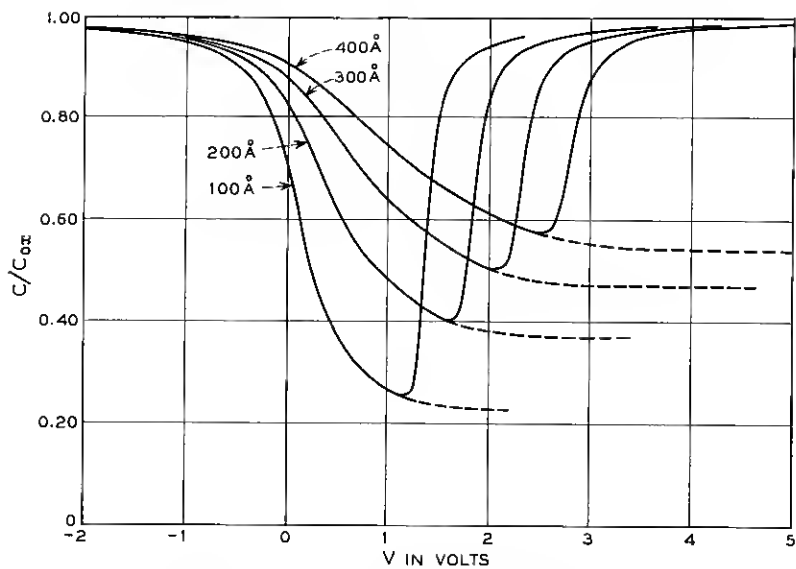


Fig. 35 — MOS capacity vs voltage. Oxide thickness 100–400 Å. ( $N_A = 1.0 \times 10^{17} \text{ cm}^{-3}$ )

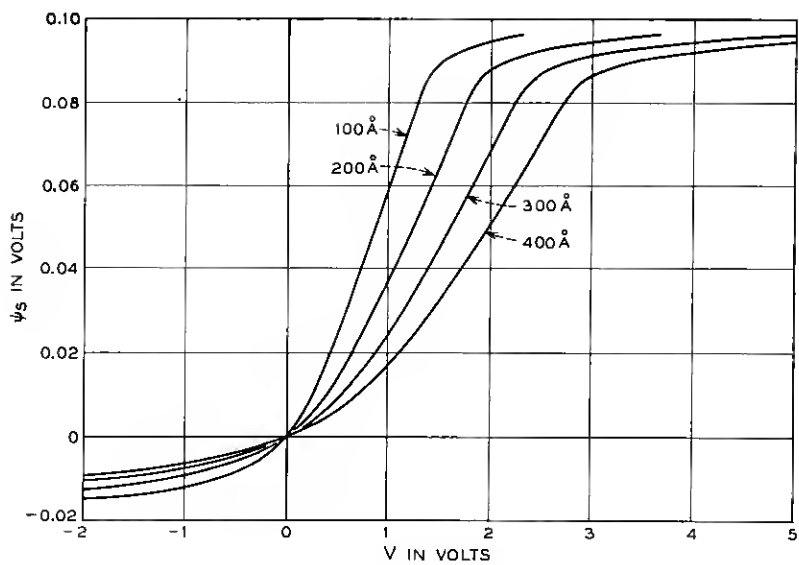


Fig. 36 — Surface potential vs voltage. Oxide thickness 100–400 Å. ( $N_A = 1.0 \times 10^{17} \text{ cm}^{-3}$ )

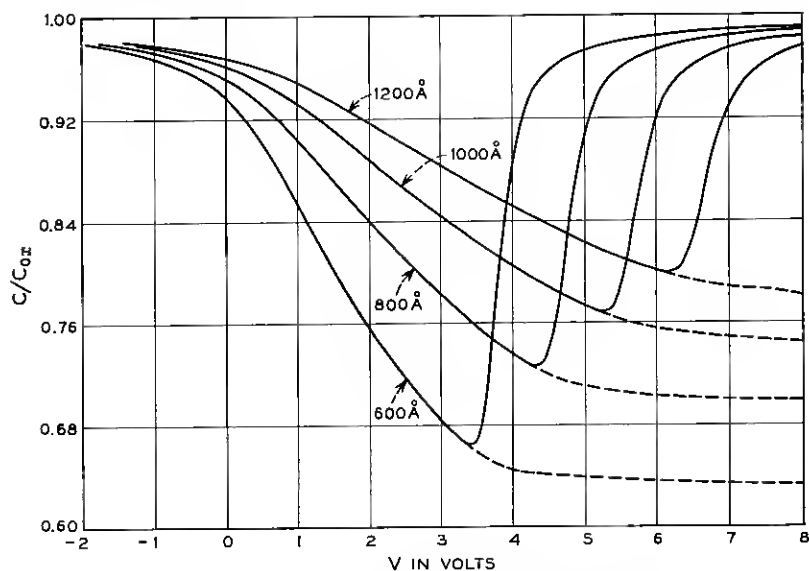


Fig. 37 — MOS capacity vs voltage. Oxide thickness 600–1200 Å. ( $N_A = 1.0 \times 10^{17} \text{ cm}^{-3}$ )

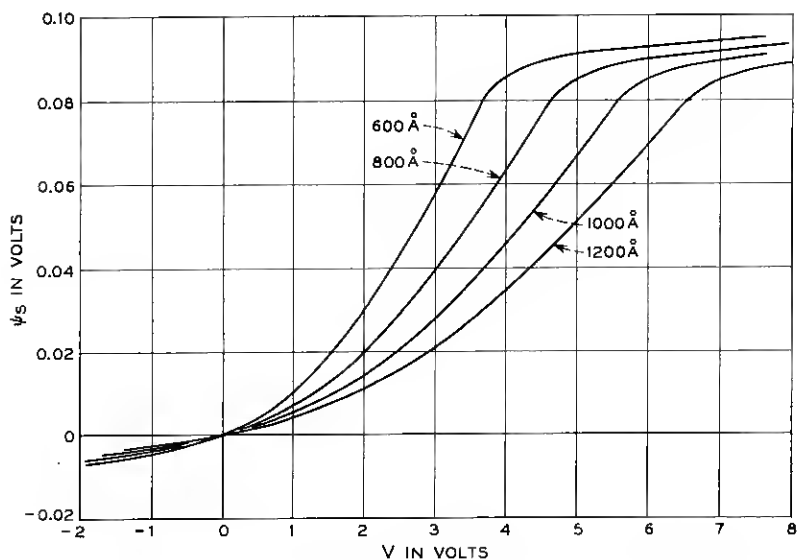


Fig. 38 — Surface potential vs voltage. Oxide thickness 600–1200 Å. ( $N_A = 1.0 \times 10^{17} \text{ cm}^{-3}$ )

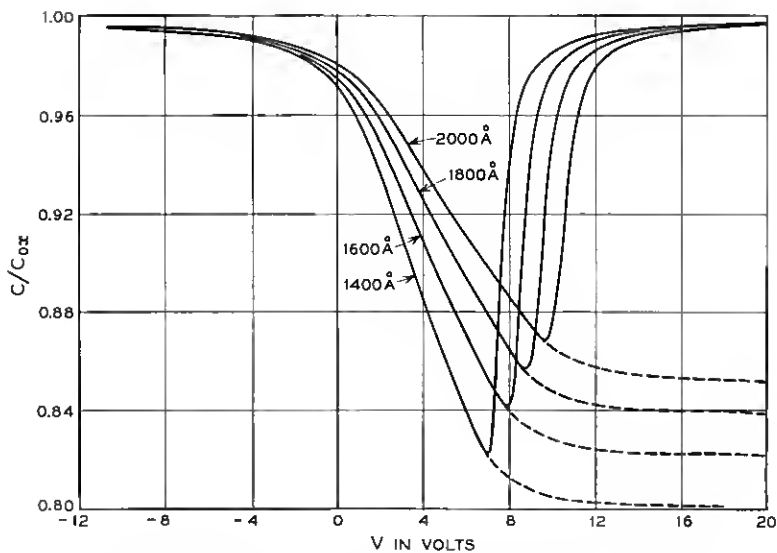


Fig. 39 — MOS capacity vs voltage. Oxide thickness 1400–2000 Å. ( $N_A = 1.0 \times 10^{17} \text{ cm}^{-3}$ )

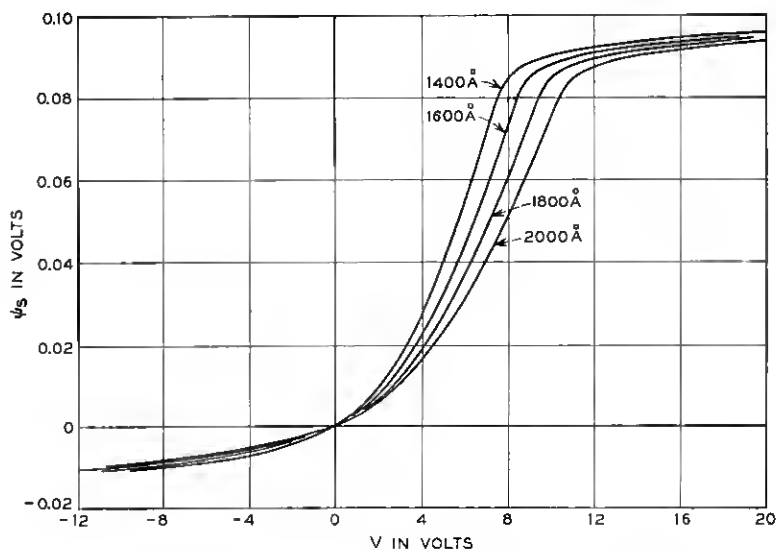


Fig. 40 — Surface potential vs voltage. Oxide thickness 1400–2000 Å. ( $N_A = 1.0 \times 10^{17} \text{ cm}^{-3}$ )

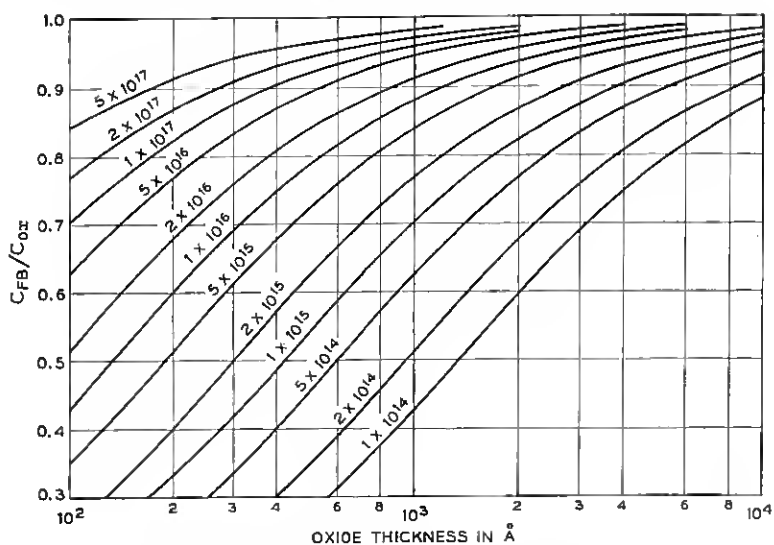


Fig. 41 — Flatband capacity vs oxide thickness I.

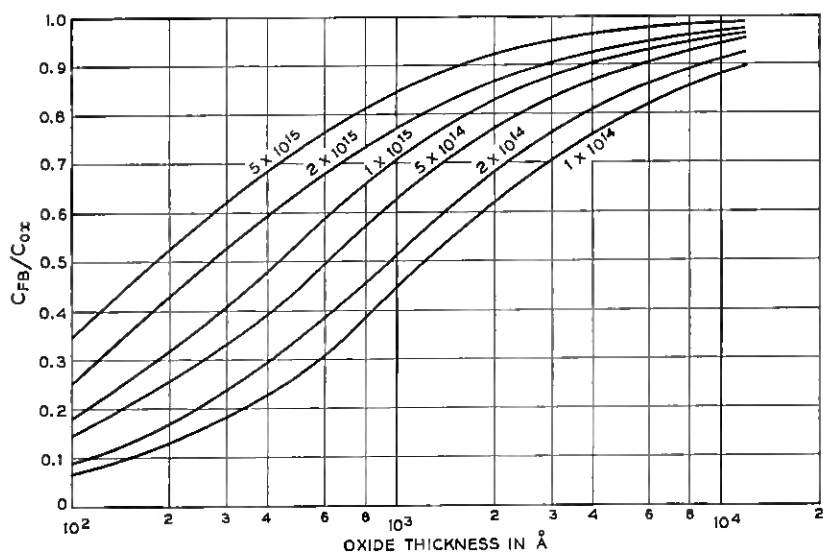


Fig. 42 — Flatband capacity vs oxide thickness II.



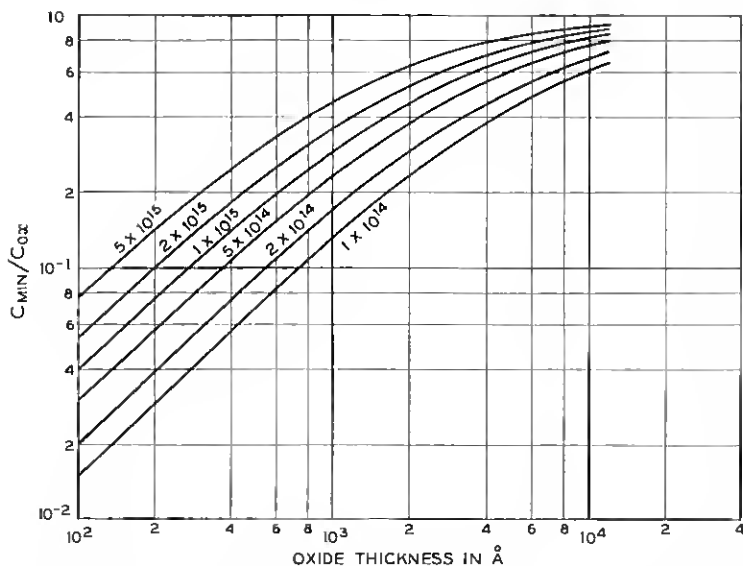


Fig. 43 — Minimum capacity vs oxide thickness. Doping density as parameter  $1 \times 10^{14}$ – $5 \times 10^{15}$  cm $^{-3}$ .

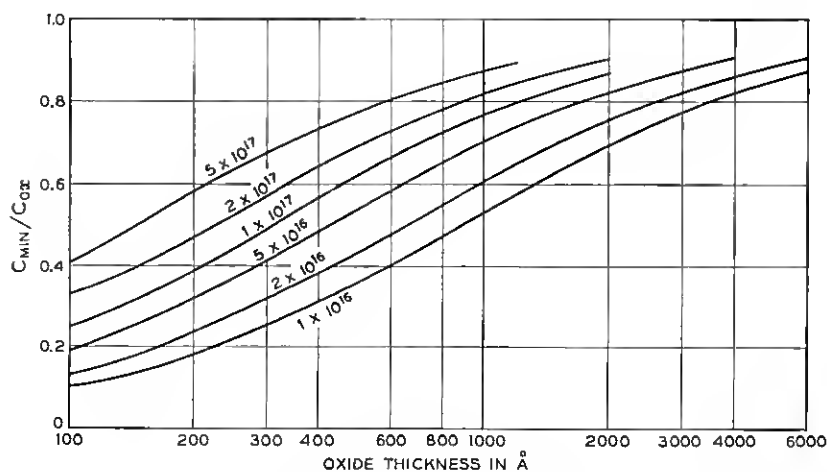


Fig. 44 — Minimum capacity vs oxide thickness. Doping density as parameter  $1 \times 10^{16}$ – $5 \times 10^{17}$  cm $^{-3}$ .

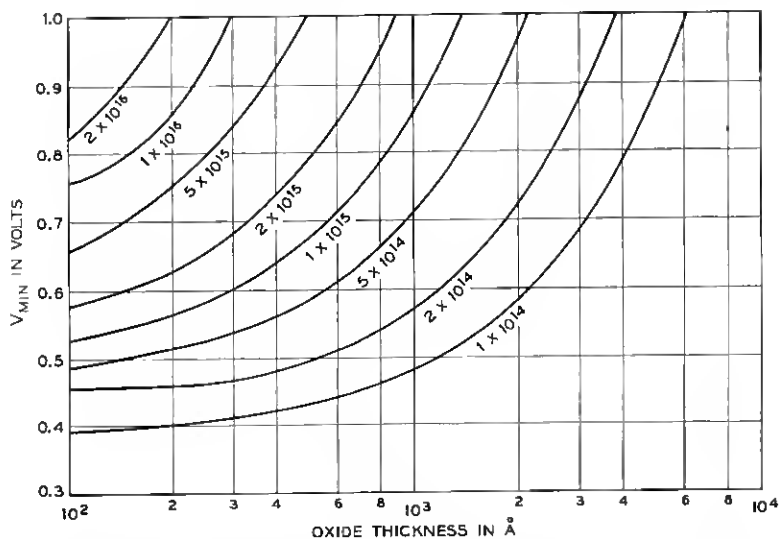


Fig. 45 — Voltage of cap. minimum vs oxide thickness. Doping density a parameter.

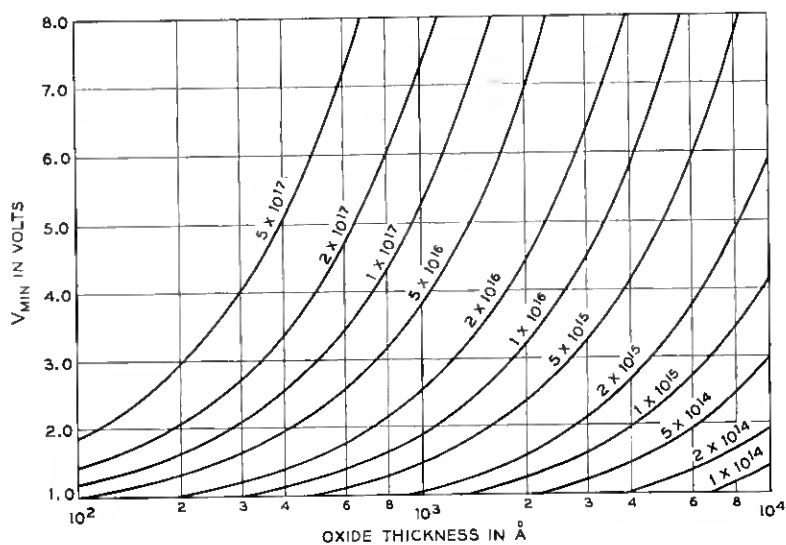


Fig. 46 — Voltage of cap. minimum vs oxide thickness. Doping density as parameter.